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DOI 10.1515/tmj-2015-0017

Intuitionistic fuzzy stability of a quadratic functional equation

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Abstract

The aim of this paper is to determine Hyers-Ulam-Rassias Stability results concerning the quadratic functional equation $f(2x+y)+f(2x-y) = 2f(x+y)+2f(x-y)+4f(x)-2f(y)$ in intuitionistic fuzzy Banach spaces.

2010 Mathematics Subject Classification. **03E72**. 97I70,39B82
Keywords. t-norm, t-conorm, Intuitionistic fuzzy normed space, Quadratic functional equation, Hyers-Ulam-Rassias stability.

1 Introduction

The study of stability problems for functional equations started with a well-known question raised by Ulam [15] in 1940 regarding the stability of group homomorphisms. In the next year Hyers [3] gave the first affirmative answer to the question of Ulam, for Cauchy functional equation under the assumption that the groups are Banach spaces and this result was generalized by T. Aoki [17] for additive mappings and by Th. M. Rassias [18] for linear mappings by considering an unbounded Cauchy difference. Gavruta [11] generalized Rassias' theorem by replacing the unbounded Cauchy difference by a general control function. F. Skof [4] generalized the Hyers-Ulam stability theorem for the function $f : X \to Y$, where X is a normed space and Y is a Banach space and then P. W. Cholewa [12] and S. Czerwik [14] extended the result of Skof. Thereafter, several stability problems of various functional equations have been studied and recently fuzzy version is also discussed.

Atanassov [8] introduced the idea of intuitionistic fuzzy sets as a generalization of fuzzy sets. A few notions of intuitionistic fuzzy metric spaces and intuitionistic fuzzy normed spaces were introduced by J. H. Park [7], Saadati and Park [13] and Samanta et.al.[19].

The stability results of many functional equations have been proved by many researchers [1, 2, 9, 10, 16, 20, 21] in fuzzy Banach spaces and intuitionistic fuzzy Banach spaces. Our interest is to established some stability results concerning the quadratic functional equation $f(2x+y)+f(2x-y) = 2f(x+y)+2f(x-y)+4f(x)-2f(y)$ in intuitionistic fuzzy Banach spaces.

2 Preliminaries

In this section we recall some lemmas, definitions and examples which will be used in this paper.

Lemma 2.1. [6] Consider the set L^* and the order relation $\leq_{L^*}$ defined by

$$L^* = \{(x_1, x_2) : (x_1, x_2) \in [0,1]^2 \text{ and } x_1 + x_2 \leq 1\},$$

Tbilisi Mathematical Journal 8(2) (2015), pp. 139–147.
Tbilisi Centre for Mathematical Sciences.

Received by the editors: 25 November 2014.
Accepted for publication: 17 June 2015.

$(x_1, x_2) \leq_{L^*} (y_1, y_2) \Leftrightarrow x_1 \leq y_1, x_2 \geq y_2, \forall (x_1, x_2), (y_1, y_2) \in L^*$.

Then $(L^*, \leq_{L^*})$ is a complete lattice. We denote its units by $0_{L^*} = (0, 1)$ and $1_{L^*} = (1, 0)$.

Definition 2.2. [8] An intuitionistic fuzzy set $A_{\zeta,\eta}$ in a universal set U is an object $A_{\zeta,\eta} = \{(\zeta_A(u), \eta_A(u)) : u \in U\}$, where $\zeta_A(u) \in [0, 1]$ and $\eta_A(u) \in [0, 1]$ for all $u \in U$ are called the membership degree and the non-membership degree respectively, of u in $A_{\zeta,\eta}$ and furthermore, satisfy $\zeta_A(u) + \eta_A(u) \leq 1$.

Definition 2.3. [5] A triangular norm (t-norm) on L^* is a mapping $\tau : (L^*)^2 \to L^*$ satisfying the following conditions :
(a) $(\forall x \in L^*)(\tau(x, 1_{L^*}) = x)$ (boundary condition);
(b) $(\forall (x, y) \in (L^*)^2)(\tau(x, y) = \tau(y, x))$ (commutativity);
(c) $(\forall (x, y, z) \in (L^*)^3)(\tau(x, \tau(y, z)) = \tau(\tau(x, y), z))$ (associativity);
(d) $(\forall (x, x', y, y') \in (L^*)^4)(x \leq_{L^*} x'$ and $y \leq_{L^*} y' \Rightarrow \tau(x, y) \leq_{L^*} \tau(x', y'))$ (monotonicity).

A t-norm τ on L^* is said to be continuous if for any $x, y \in L^*$ and any sequences $\{x_n\}$ and $\{y_n\}$ which converge to x and y respectively,

$$\lim_{n \to \infty} \tau(x_n, y_n) = \tau(x, y).$$

For example, let $a = (a_1, a_2), b = (b_1, b_2) \in L^*$, consider
$\tau(a, b) = (a_1 b_1, \min\{a_2 + b_2, 1\})$ and $M(a, b) = (\min\{a_1, b_1\}, \max\{a_2, b_2\})$.
Then $\tau(a, b)$ and $M(a, b)$ are continuous t-norm.

Now, we define a sequence τ^n recursively by $\tau^1 = \tau$ and

$$\tau^n\left(x^{(1)}, \cdots, x^{(n+1)}\right) = \tau\left(\tau^{n-1}\left(x^{(1)}, \cdots, x^{(n)}\right), x^{(n+1)}\right)$$

for all $n \geq 2$ and $x^{(i)} \in L^*$.

Definition 2.4. [5] A continuous t-norm τ on L^* is said to be continuous t-representable if there exists a continuous t-norm $*$ and a continuous t-conorm $\diamond$ on $[0, 1]$ such that, for all $x = (x_1, x_2), y = (y_1, y_2) \in L^*$,

$$\tau(x, y) = (x_1 * y_1, x_2 \diamond y_2).$$

Definition 2.5. [5] A negator on L^* is any decreasing mapping $N : L^* \to L^*$ satisfying $N(0_{L^*}) = 1_{L^*}$ and $N(1_{L^*}) = 0_{L^*}$. If $N(N(x)) = x$ for all $x \in L^*$, then N is called an involutive negator. A negator on $[0, 1]$ is a decreasing mapping $N : [0, 1] \to [0, 1]$ satisfying $N(0) = 1$ and $N(1) = 0$. N_s denotes the standard negator on $[0, 1]$ defined by $N_s(x) = 1 - x$ for all $x \in [0, 1]$.

Definition 2.6. [16] (1) Let $L = (L^*, \leq_{L^*})$. The triple (X, P, τ) is said to be an L-fuzzy normed space if X is a vector space, τ is a continuous t-norm on L^* and P is an L-fuzzy set on $X \times (0, +\infty)$ satisfying the following conditions for all $x, y \in X$ and $t, s > 0$,
(a) $P(x, t) > 0_{L^*}$;
(b) $P(x, t) = 1_{L^*}$ if and only if $x = 0$;

(c) $P(\alpha x,\, t) \,=\, P\left(x,\, \frac{t}{|\alpha|}\right)$ for all $\alpha \neq 0$;
(d) $P(x + y,\, t + s) \geq_{L^*} \tau(P(x,\, t),\, P(y,\, s))$;
(e) $P(x,\, .) : (0,\, \infty) \to L^*$ is continuous;
(f) $\lim_{t \to 0} P(x,\, t) = 0_{L^*}$ and $\lim_{t \to \infty} P(x,\, t) = 1_{L^*}$.

In this case P is called an L-fuzzy norm (briefly, L^*-fuzzy norm).
(2) If $P = P_{\mu,\nu}$ is an intuitionistic fuzzy set, then the triple $(X,\, P_{\mu,\nu},\, \tau)$ is said to be an intuitionistic fuzzy normed space (briefly, IFN-space). In this case $P = P_{\mu,\nu}$ is called an intuitionistic fuzzy norm on X.
Note that, if P is an L^*-fuzzy norm on X, then the following are satisfied :
(i) $P(x,\, t)$ is nondecreasing with respect to t for all $x \in X$.
(ii) $P(x - y,\, t) = P(y - x,\, t)$ for all $x,\, y \in X$ and $t > 0$.

Example 2.7. Let $(X,\, \|.\|)$ be a normed space.
Let $\tau(a,\, b) = (a_1 b_1,\, min\{a_2 + b_2,\, 1\})$ for all $a = (a_1,\, a_2),\, b = (b_1,\, b_2) \in L^*$ and $\mu,\, \nu$ be membership and non-membership degree of an intuitionistic fuzzy set defined by

$$P_{\mu,\nu}(x,\, t) \,=\, (\mu_x(t),\, \nu_x(t)) \,=\, \left(\frac{t}{t + m\|x\|},\, \frac{\|x\|}{t + \|x\|}\right)$$

for all $t \in \mathbb{R}^+$ in which $m > 1$. Then $(X,\, P_{\mu,\nu},\, \tau)$ is an IFN-space. Here, $\mu(x,\, t) + \nu(x,\, t) = 1$ for $x = 0$ and $\mu(x,\, t) + \nu(x,\, t) < 1$ for $x \neq 0$.
Let $M(a,\, b) = (\min\{a_1,\, b_1\},\, \max\{a_2,\, b_2\})$ for all
$a = (a_1,\, a_2),\, b = (b_1,\, b_2) \in L^*$ and $\mu,\, \nu$ be membership and non-membership degree of an intuitionistic fuzzy set defined by

$$P_{\mu,\nu}(x,\, t) \,=\, (\mu_x(t),\, \nu_x(t)) \,=\, \left(e^{-\frac{\|x\|}{t}},\, e^{-\frac{\|x\|}{t}}\left(e^{\frac{\|x\|}{t}} - 1\right)\right)$$

for all $t \in \mathbb{R}^+$. Then $(X,\, P_{\mu,\nu},\, M)$ is an IFN-space.

Definition 2.8. (1) A sequence $\{x_n\}$ in an IFN-space $(X,\, P_{\mu,\nu},\, \tau)$ is said to be convergent to a point $x \in X$ (denoted by $x_n \to x$) if $P_{\mu,\nu}(x_n - x,\, t) \to 1_{L^*}$ as $n \to \infty$ for every $t > 0$.
(2) A sequence $\{x_n\}$ in an IFN-space $(X,\, P_{\mu,\nu},\, \tau)$ is said to be a Cauchy sequence if, for any $0 < \varepsilon < 1$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that

$$P_{\mu,\nu}(x_n - x_m,\, t) >_{L^*} (N_s(\varepsilon),\, \varepsilon)$$

for all $n,\, m \geq n_0$, where N_s is the standard negator.
(3) An IFN-space $(X,\, P_{\mu,\nu},\, \tau)$ is said to be complete if every Cauchy sequence in $(X,\, P_{\mu,\nu},\, \tau)$ is convergent in $(X,\, P_{\mu,\nu},\, \tau)$. A complete intuitionistic fuzzy normed space is called an intuitionistic fuzzy Banach space.

3 Stability of The Functional Equation

Throughout this section, we assume that $X,\, Y,\, Z$ are real vector spaces.

Theorem 3.1. Let $\psi : X^2 \to Z$ be a mapping such that

$$P'_{\mu,\nu}(\psi(2x,\, 2y),\, t) \;\geq_{L^*}\; P'_{\mu,\nu}(\alpha\, \psi(x,\, y),\, t) \tag{3.1}$$

for some α satisfying $0 < \alpha < 4$ and for all $x, y \in X, t > 0$, where $(Z, P'_{\mu,\nu}, \tau)$ is an IFN-space. Let $(Y, P_{\mu,\nu}, \tau)$ be a complete IFN-space and $f : X \to Y$ be a mapping such that

$$P_{\mu,\nu}(f(2x + y) + f(2x - y) - 2f(x + y) - 2f(x - y) - 4f(x) + 2f(y), t) \tag{3.2}$$

$$\geq_{L^*} P'_{\mu,\nu}(\psi(x, y), t)$$

for all $x, y \in X$ and $t > 0$. Then there exists a unique quadratic mapping $Q : X \to Y$ such that

$$P_{\mu,\nu}(f(x) - Q(x), t) \geq_{L^*} P'_{\mu,\nu}(\psi(x, 0), 2(4 - \alpha)t) \tag{3.3}$$

and

$$\frac{f(2^n x)}{4^n} \to Q(x), \quad as \;\; n \to \infty \tag{3.4}$$

for all $x \in X$, $t > 0$.

Proof. Putting $x = y = 0$ in $f(2x + y) + f(2x - y) = 2f(x + y) + 2f(x - y) + 4f(x) - 2f(y)$, we obtain $f(0) = 0$. Again, putting $y = 0$ in (3.2), we get for all $x \in X$, $t > 0$,

$$P_{\mu,\nu}(f(x) - 4^{-1}f(2x), t) \geq_{L^*} P'_{\mu,\nu}(\psi(x, 0), 8t). \tag{3.5}$$

Replacing x by $2^n x$ in (3.5) and using (3.1), we have for all $x \in X$, $t > 0, n \in \mathbb{N}$,

$$P_{\mu,\nu}\left(\frac{f(2^n x)}{4^n} - \frac{f(2^{n+1} x)}{4^{n+1}}, t\right) \geq_{L^*} P'_{\mu,\nu}\left(\psi(x, 0), 8\left(\frac{4}{\alpha}\right)^n t\right). \tag{3.6}$$

Now, for all $x \in X$, $t > 0, n \in \mathbb{N}$ we get

$$P_{\mu,\nu}\left(f(x) - \frac{f(2^n x)}{4^n}, \frac{t}{8}\sum_{i=0}^{n-1}\left(\frac{\alpha}{4}\right)^i\right)$$

$$\geq_{L^*} \tau^{n-1}\left(P_{\mu,\nu}\left(f(x) - \frac{f(2x)}{4}, \frac{t}{8}\right), P_{\mu,\nu}\left(\frac{f(2x)}{4} - \frac{f(2^2 x)}{4^2}, \frac{t}{8}\frac{\alpha}{4}\right),\right.$$

$$\left. \cdots, P_{\mu,\nu}\left(\frac{f(2^{n-1} x)}{4^{n-1}} - \frac{f(2^n x)}{4^n}, \frac{t}{8}\left(\frac{\alpha}{4}\right)^{n-1}\right)\right)$$

$$\geq_{L^*} P'_{\mu,\nu}(\psi(x, 0), t) \quad [\text{by } (3.5), (3.6)].$$

It implies that for all $x \in X$, $t > 0$ and $n \in \mathbb{N}$,

$$P_{\mu,\nu}\left(f(x) - \frac{f(2^n x)}{4^n}, t\right) \geq_{L^*} P'_{\mu,\nu}\left(\psi(x, 0), \frac{8t}{\sum_{i=0}^{n-1}\left(\frac{\alpha}{4}\right)^i}\right). \tag{3.7}$$

Replacing x by $2^p x$ in (3.7) and using (3.1), we get for all $x \in X$, $t > 0$ and $n, p \in \mathbb{N}$,

$$P_{\mu,\nu}\left(\frac{f(2^p x)}{4^p} - \frac{f(2^{n+p}x)}{4^{n+p}}, t\right) \geq_{L^*} P'_{\mu,\nu}\left(\psi(x,0), \frac{8t}{\left(\frac{\alpha}{4}\right)^p \sum_{i=0}^{n-1}\left(\frac{\alpha}{4}\right)^i}\right). \tag{3.8}$$

Taking limit as $p \to \infty$, we get for all $x \in X$, $n \in \mathbb{N}$ and $t > 0$,

$$P_{\mu,\nu}\left(\frac{f(2^p x)}{4^p} - \frac{f(2^{n+p}x)}{4^{n+p}}, t\right) \to 1_{L^*} \ \ as \ \ p \to \infty.$$

Therefore the sequence $\left\{\frac{f(2^n x)}{4^n}\right\}$ is a Cauchy sequence in $(Y, P_{\mu,\nu}, \tau)$. Since $(Y, P_{\mu,\nu}, \tau)$ is a complete IFN-space, there exists some function $Q : X \to Y$ such that (3.4) holds. Let $\delta > 0$. Now, for all $x \in X$, $t > 0$ and $n \in \mathbb{N}$,

$$P_{\mu,\nu}(f(x) - Q(x), t + \delta)$$

$$\geq_{L^*} \tau\left(P'_{\mu,\nu}\left(\psi(x,0), \frac{8t}{\sum_{i=0}^{n-1}\left(\frac{\alpha}{4}\right)^i}\right), P_{\mu,\nu}\left(\frac{f(2^n x)}{4^n} - Q(x), \delta\right)\right) \quad [\text{by } (3.7)].$$

Taking limit as $n \to \infty$, we get for all $x \in X$, $t > 0$,

$$P_{\mu,\nu}(f(x) - Q(x), t + \delta) \geq_{L^*} \tau\left(P'_{\mu,\nu}\left(\psi(x,0), \frac{8t}{\sum_{i=0}^{\infty}\left(\frac{\alpha}{4}\right)^i}\right), 1_{L^*}\right) \quad [\text{by } (3.4)].$$

Letting $\delta \to 0$, we get (3.3). From definition of Q we get for all $x \in X$ and $n \in \mathbb{N}$,

$$Q(2^n x) = 4^n Q(x). \tag{3.9}$$

We replace x and y by $2^n x$ and $2^n y$ respectively in (3.2) to get for all $x, y \in X$, $t > 0$ and $n \in \mathbb{N}$,

$$P_{\mu,\nu}\left(\frac{f(2^n(2x+y))}{4^n} + \frac{f(2^n(2x-y))}{4^n} - \frac{2f(2^n(x+y))}{4^n} - \frac{2f(2^n(x-y))}{4^n} - \frac{4f(2^n x)}{4^n} + \frac{2f(2^n y)}{4^n}, t\right)$$

$$\geq_{L^*} P'_{\mu,\nu}(\psi(2^n x, 2^n y), 4^n t)$$

$$\geq_{L^*} P'_{\mu,\nu}\left(\psi(x,y), \left(\frac{4}{\alpha}\right)^n t\right) \quad [\text{by } (3.1)].$$

Taking limit $n \to \infty$, we get for all $x, y \in X$, $t > 0$,

$$P_{\mu,\nu}(Q(2x + y) + Q(2x - y) - 2Q(x + y) - 2Q(x - y) - 4Q(x) + 2Q(y), t) = 1_{L^*}.$$

It implies that for all $x, y \in X$,

$$Q(2x + y) + Q(2x - y) - 2Q(x + y) - 2Q(x - y) - 4Q(x) + 2Q(y) = 0.$$

This shows that Q is quadratic. To prove the uniqueness we assume that $Q' : X \to Y$ is a quadratic function satisfying (3.3) and (3.4). Now, using (3.9), (3.3) and (3.1), we get for all $x \in X$, $t > 0$ and $n \in \mathbb{N}$,

$$P_{\mu,\nu}(Q(x) - Q'(x), t)$$

$$\geq_{L^*} \tau\left(P_{\mu,\nu}\left(f(2^n x) - Q(2^n x), \frac{4^n t}{2}\right), P_{\mu,\nu}\left(f(2^n x) - Q'(2^n x), \frac{4^n t}{2}\right)\right)$$

$$\geq_{L^*} \tau\left(P'_{\mu,\nu}\left(\psi(x, 0), \frac{4^n (4 - \alpha)t}{\alpha^n}\right), P'_{\mu,\nu}\left(\psi(x, 0), \frac{4^n (4 - \alpha)t}{\alpha^n}\right)\right).$$

Taking limit as $n \to \infty$, we get for all $x \in X$, $t > 0$,

$$P_{\mu,\nu}(Q(x) - Q'(x), t) = 1_{L^*}.$$

It implies that $Q(x) = Q'(x)$ for all $x \in X$. This proves that Q is unique. This completes the proof. Q.E.D.

Corollary 3.2. Let $(Y, P_{\mu,\nu}, \tau)$ be a complete IFN-space and $(Z, P'_{\mu,\nu}, \tau)$ be an IFN-space. Let p, q be two non-negative real numbers less than 1 and $z_0 \in Z$. Let $f : X \to Y$ be a mapping such that

$$P_{\mu,\nu}(f(2x + y) + f(2x - y) - 2f(x + y) - 2f(x - y) - 4f(x) + 2f(y), t)$$
$$\geq_{L^*} P'_{\mu,\nu}((\|x\|^p + \|y\|^q)z_0, t)$$

for all $x, y \in X$ and $t > 0$. Then there exists a unique quadratic mapping $Q : X \to Y$ such that

$$P_{\mu,\nu}(f(x) - Q(x), t) \geq_{L^*} P'_{\mu,\nu}(\|x\|^p z_0, 2(4 - 4^p)t)$$

and

$$\frac{f(2^n x)}{4^n} \to Q(x), \quad as \ n \to \infty$$

for all $x \in X$, $t > 0$.

Proof. Define $\psi(x, y) = (\|x\|^p + \|y\|^q)z_0$ and take $\alpha = 4^p$. Clearly, (3.1) is satisfied and $0 < \alpha < 4$. Thus the theorem (3.1) completes the proof. Q.E.D.

Corollary 3.3. Let $(Y, P_{\mu,\nu}, \tau)$ be a complete IFN-space and $(Z, P'_{\mu_1,\nu_1}, \tau)$ be an IFN-space. Let $z_0 \in Z$, $\varepsilon \geq 0$ and $f : X \to Y$ be a mapping such that

$$P_{\mu,\nu}(f(2x+y) + f(2x-y) - 2f(x+y) - 2f(x-y) - 4f(x) + 2f(y), t)$$
$$\geq_{L^*} P'_{\mu,\nu}(\varepsilon z_0, t)$$

for all $x, y \in X$ and $t > 0$. Then there exists a unique quadratic mapping $Q : X \to Y$ such that

$$P_{\mu,\nu}(f(x) - Q(x), t) \geq_{L^*} P'_{\mu,\nu}(\varepsilon z_0, t)$$

and

$$\frac{f(2^n x)}{4^n} \to Q(x), \quad as \ \ n \to \infty$$

for all $x \in X$, $t > 0$.

Proof. Define $\psi(x, y) = \varepsilon z_0$ and take $\alpha = 3.5$. Clearly, (3.1) is satisfied. Thus the theorem (3.1) completes the proof. Q.E.D.

Theorem 3.4. Let $\psi : X^2 \to Z$ be a mapping such that

$$P'_{\mu,\nu}\left(\psi\left(\frac{x}{2}, \frac{y}{2}\right), t\right) \geq_{L^*} P'_{\mu,\nu}\left(\frac{1}{\alpha}\psi(x, y), t\right)$$

for some $\alpha > 4$ and for all $x, y \in X$, $t > 0$, where $(Z, P'_{\mu,\nu}, \tau)$ is an IFN-space. Let $(Y, P_{\mu,\nu}, \tau)$ be a complete IFN-space and $f : X \to Y$ be a mapping such that

$$P_{\mu,\nu}(f(2x+y) + f(2x-y) - 2f(x+y) - 2f(x-y) - 4f(x) + 2f(y), t)$$
$$\geq_{L^*} P'_{\mu,\nu}(\psi(x, y), t)$$

for all $x, y \in X$ and $t > 0$. Then there exists a unique quadratic mapping $Q : X \to Y$ such that

$$P_{\mu,\nu}(f(x) - Q(x), t) \geq_{L^*} P'_{\mu,\nu}(\psi(x, 0), 2(\alpha - 4)t)$$

and

$$4^n f\left(\frac{x}{2^n}\right) \to Q(x), \quad as \ \ n \to \infty$$

for all $x \in X$, $t > 0$.

References

[1] A. K. Mirmostafaee, M. S. Moslehian, *Fuzzy versions of Hyers-Ulam-Rassias theorem*, Fuzzy Sets and Systems, 159 (2008), 720729.

[2] C. Park, *Fuzzy stability of a functional equation associated with inner product space*, Fuzzy Sets and Systems, 160 (2009), 16321642.

[3] D. H. Hyers, *On the stability of the linear functional equation*, Proc. Nat. Acad. Sci. U.S.A., 27 (1941), 222−224.

[4] F. Skof, *Proprieta locali e approssimazione di opratori*, Rend. Sem. Mat. Fis. Milano, 53 (1983), 113−129.

[5] G. Deschrijver, C. Cornelis and E. E. Kerre, *On the representation of intuitionistic fuzzy t-norms and t-conorms*, IEEE Transaction on Fuzzy Systems, 12 (2004), 45−61.

[6] G. Deschrijver, E. E. Kerre, *On the relationship between some extensions of fuzzy set theory*, Fuzzy Sets and Systems, 23 (2003), 227−235.

[7] J. H. Park, *Intuitionistic fuzzy metric spaces*, Chaos, Solitons and Fractals, 22 (2004), 1039−1046.

[8] K. T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, 20 (1986), 87−96.

[9] N. Chandra Kayal, P. Mondal and T. K. Samanta *The Generalized Hyers - Ulam - Rassias Stability of a Quadratic Functional Equation in Fuzzy Banach Spaces*, Journal of New Results in Science, 1 (5) (2014), 83−95.

[10] N. Chandra Kayal, P. Mondal and T. K. Samanta, *The Fuzzy Stability of a Pexiderized Functional Equation*, Mathematica Moravica, 18 (2) (2014), 1−14.

[11] P. Gavruta, *A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings*, J. Math. Anal. appl., 184 (1994), 431−436.

[12] P. W. Cholewa, *Remarks on the stability of functional equations*, Aequationes Math., 27 (1984), 76−86.

[13] R. Saadati, J. H. Park, *On the intuitionistic fuzzy topological spaces*, Chaos, Solitons and Fractals, 27 (2006), 331−344.

[14] S. Czerwik, *On the stability of the quadratic mappings in normed spaces*, Abh. Math. Sem. Univ. Hamburg, 62 (1992), 59−64.

[15] S. M. Ulam, *Problems in Modern Mathematics*, Chapter VI, Science Editions, Wiley, New York, 1960.

[16] S. Shakeri, *Intutionistic fuzzy stability of Jenson type mapping*, J. Non linear Sc. Appl., 2 (2009), no.-2, 105−112.

[17] T. Aoki, *On the Stability of Linear Transformation in Banach Spaces*, J. Math. Soc. Japan, 2 (1950), 64−66.

[18] Th. M. Rassias, *On the stability of the linear mapping in Banach space*, Proc. Amer. Mathematical Society, 72(2) (1978), 297−300.

[19] T. K. Samanta and Iqbal H. Jebril, *Finite dimentional intuitionistic fuzzy normed linear space*, Int. J. Open Problems Compt. Math., 2(4) (2009), 574591.

[20] T. K. Samanta, P. Mondal and N. Chandra Kayal, *The generalized Hyers-Ulam-Rassias stability of a quadratic functional equation in fuzzy Banach spaces*, Annals of Fuzzy Mathematics and Informatics, Vol. 6, No. 2, (2013), 285294.

[21] T. K. Samanta, N. Chandra Kayal and P. Mondal, *The Stability of a General Quadratic Functional Equation in Fuzzy Banach Space*, Journal of Hyperstructures, 1 (2) (2012), 71−87.